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## LETTER TO THE EDITOR

## Model of the mixed state of type-II superconductors in high magnetic fields

I L Landau<sup>1,2</sup> and H R Ott<sup>1</sup><sup>1</sup> Laboratorium für Festkörperphysik, ETH Hönggerberg, CH-8093 Zürich, Switzerland<sup>2</sup> Kapitza Institute for Physical Problems, 117334 Moscow, Russia

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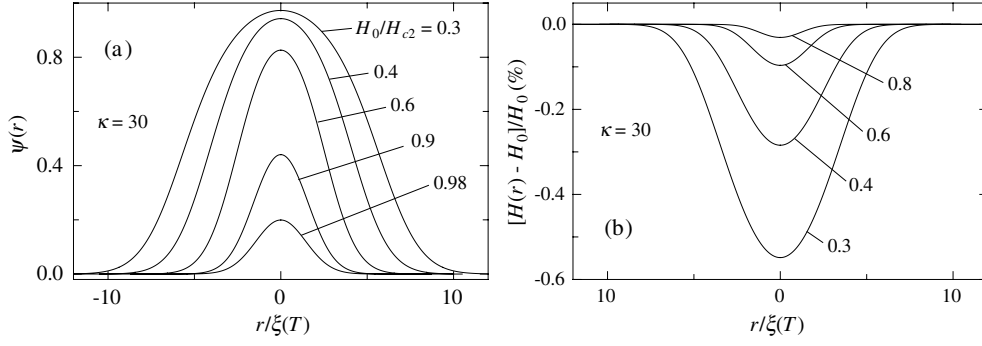
Online at [stacks.iop.org/JPhysCM/14/L313](http://stacks.iop.org/JPhysCM/14/L313)**Abstract**

In superconductors with large values of the Ginzburg–Landau parameter  $\kappa$ , exposed to magnetic fields close to the upper critical field  $H_{c2}$ , the magnetic field is practically homogeneous across the sample and the density of supercurrents is negligibly small. In this case, there is no obvious reason for the formation of Abrikosov vortices, characteristic for the well known mixed state. We consider an alternative model for describing the mixed state for  $\kappa \gg 1$  and magnetic fields close to  $H_{c2}$ . We argue that with decreasing magnetic field the traditional vortex structure is adopted via a first-order phase transition, revealed by discontinuities in the magnetization as well as the resistivity.

(Some figures in this article are in colour only in the electronic version)

It is commonly accepted that the mixed state of a type-II superconductor is characterized by the penetration of an external magnetic field into the sample along quantized vortex lines or vortices. The spatial extent of the superconducting order parameter  $\psi(r)$  near a vortex axis is determined by circular currents flowing around the vortex line. The velocity of the relevant charge carriers, i.e., the Cooper pairs, diverges at the centre of the vortex line (see, e.g., [1]), thus reducing the superconducting order parameter progressively until it vanishes on the vortex axis. In most cases the vortices may thus be considered as thin normal filaments embedded in a superconducting environment. A completely different situation may, however, be established in superconductors with a Ginzburg–Landau (GL) parameter  $\kappa \gg 1$ . If the applied magnetic field is close to the upper critical field  $H_{c2}$ , the distance between adjacent vortex cores is much smaller than the magnetic field penetration depth  $\lambda(T)$ . In this case, there is practically no expulsion of the magnetic field from superconducting regions and the density of shielding currents is negligibly small. These circumstances are naturally unfavourable for the formation of common Abrikosov vortices and hence the character of the mixed state in magnetic fields close to  $H_{c2}$  may be very different from that adopted at lower fields.

In this letter we consider that in magnetic fields close to  $H_{c2}$ , the natural alternative to the conventional vortex structure is the formation of superconducting filaments, embedded in



**Figure 1.** The radial variation of (a) the normalized order parameter  $\psi(r)$  and (b) the magnetic field for different values of  $H_0/H_{c2}$ .  $\xi(T) = \lambda(T)/\kappa$  is the GL coherence length.

the matrix of the normal metal. The properties of such superconducting filaments may be analysed by numerically solving the GL equations<sup>3</sup>. First, we consider a single cylindrical superconducting filament in an infinitely extended normal metal. The magnetic field inside the normal metal is oriented parallel to the filament and its value is set to  $H_0 < H_{c2}$ . For the numerical analysis we have chosen cylindrical coordinates  $(r, \phi, z)$  with the  $z$ -axis parallel to the filament and  $r = 0$  at its centre. The GL equations for an infinitely long cylindrical filament

$$-\frac{1}{\kappa^2} \left( \frac{d^2 \psi}{dr^2} + \frac{1}{r} \frac{d\psi}{dr} \right) + (A^2 - 1)\psi + \psi^3 = 0 \quad (1)$$

and

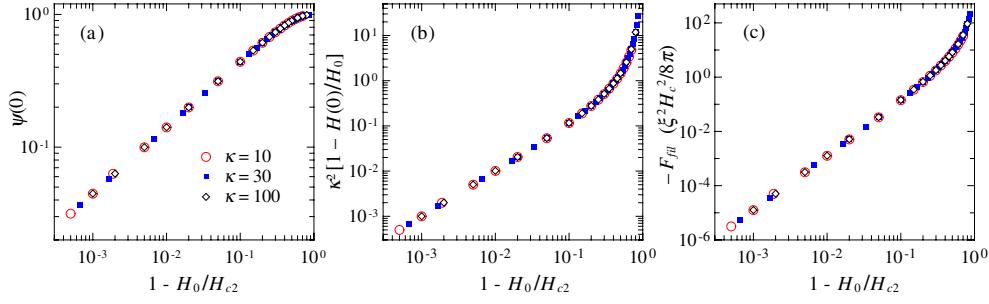
$$\frac{d}{dr} \left( \frac{dA}{dr} + \frac{A}{r} \right) = A\psi^2 \quad (2)$$

may thus be solved in one dimension, with all quantities depending only on the radial coordinate  $r$ . Here,  $\psi$  is the superconducting order parameter normalized by its equilibrium value in a bulk superconductor;  $A$  is the vector potential of the magnetic field expressed in units of  $\sqrt{2}H_c\lambda(T)$ , where  $H_c$  is the thermodynamic critical field. The coordinate  $r$  is measured in units of  $\lambda(T)$ .

The boundary conditions are set at the centre of the filament ( $r = 0$ ). For the superconducting order parameter  $\psi(r)$ , we have  $d\psi/dr|_{r=0} = 0$  and  $\psi(0) = \text{const}$ . For the vector potential  $A(r)$ , we have  $A(0) = 0$  and  $\text{curl } A|_{r=0} = H(0)$ , the value of the magnetic field at the centre of the filament. We have checked that the solutions with  $A(0) = 0$ , equivalent to a vanishing supercurrent at  $r = 0$ , correspond to a minimum of the free energy. By integration of the GL equations with these boundary conditions, we obtain  $\psi(r)$ ,  $A(r)$  and the distribution of the magnetic field  $H(r)$  around the filament. We also calculated the Gibbs free energy  $F_{fil}$  of the filament per unit length. For a single filament,  $F_{fil}$  reaches its minimum if, at the boundary with the normal metal, the order parameter vanishes with a zero derivative, i.e.,  $\psi(r) \rightarrow 0$  and  $d\psi/dr \rightarrow 0$  for  $r \rightarrow \infty$ .

The calculations have been made for three values of  $\kappa$ : 10, 30, and 100. Figure 1(a) shows the calculated profiles of the normalized superconducting order parameter  $\psi(r)$  for one filament and, in figure 1(b), corresponding profiles of the magnetic field have been plotted. For all  $\kappa$ -values between 10 and 100 and  $H_0/H_{c2} \geq 0.3$ , these profiles are independent of  $\kappa$ .

<sup>3</sup> The calculation procedure is described in [2]. The only difference is that here we use the GL equations written in cylindrical coordinates.



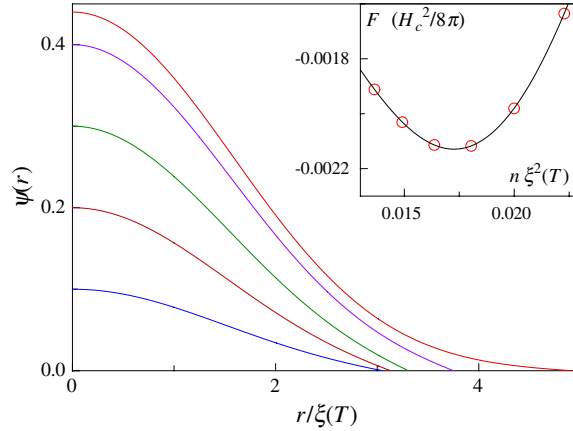
**Figure 2.** Different parameters of a single superconducting filament as functions of  $(1 - H_0/H_{c2})$ . (a) The amplitude of the normalized superconducting order parameter at the centre of the filament. (b) The normalized difference between the magnetic field at the centre of the filament and  $H_0$ , multiplied by  $\kappa^2$ . (c) The Gibbs free energy of the filament  $F_{fil}$  (per unit of length) in units of  $\xi^2(T)H_c^2/8\pi$  with respect to that of the normal metal in a magnetic field equal to  $H_0$ .

In this approach, all characteristics of the filament are uniquely determined by the value of the applied magnetic field  $H_0$ .

In figures 2(a)–(c) we display various quantities, plotted versus  $(1 - H_0/H_{c2})$ . Figure 2(a) shows the amplitude of the normalized GL order parameter at the centre of the filament; in figure 2(b) we have plotted the product  $\kappa^2[1 - H(0)/H_0]$ , and in figure 2(c) we show the free energy of the filament  $F_{fil}$ . It may be seen that while  $\psi(0)$  and  $F_{fil}$  are independent of  $\kappa$ , the expulsion of the magnetic field from the centre of the filament,  $[1 - H(0)/H_0]$ , is inversely proportional to  $\kappa^2$ , i.e., the magnetic moment of the filament rapidly decreases with increasing  $\kappa$ . According to figure 2(c) the free energy of the filament with respect to that of the normal metal is negative and it vanishes at  $H_0 = H_{c2}$ . This obvious but gratifying result demonstrates that the superconducting phase can only be stable in magnetic fields below  $H_{c2}$ .

So far, we have considered a single superconducting filament in a normal-metal matrix and we now consider the interaction between the filaments. Because the current density in the normal metal between the filaments is zero, there is no long-range interaction between them and the contribution to the free energy due to filament–filament interaction is zero unless the neighbouring filaments are very close to each other. In order to show that the short-range interaction between the filaments is repulsive, we introduce the velocity of superconducting electrons  $v_s = j_s/(2e\psi^2)$ , where  $j_s$  is the supercurrent,  $c$  is the speed of light, and  $e$  is the electron charge. In our case  $v_s$  increases with the distance from the axis of the filament approximately proportionally to  $r$ . The velocities  $v_s$  arising from the neighbouring filaments have opposite directions in the space between them. Because  $v_s$  cannot have discontinuities in the superconducting phase, the filaments cannot merge, but must always be divided by a boundary where the order parameter<sup>4</sup>  $\psi \equiv 0$ . The situation in the boundary region is very similar to that arising in the centre of the Abrikosov vortex where  $v_s$  also changes its sign, requiring that  $\psi = 0$  along the vortex axis.

<sup>4</sup> The fact that superconducting domains in the mixed state of the type-II superconductor cannot merge into one may be even more clearly demonstrated if we consider two superconducting planes instead of filaments. We choose the vector potential  $A = 0$  along the boundary between the planes and the order parameter in its complex form. In this case  $v_s$  is proportional to  $\nabla\phi$ , where  $\phi$  is the phase of the order parameter. Because  $\nabla\phi$  changes its sign if we go from one superconducting plane to the other, it is clear that the order parameters of these two planes cannot be forced to merge. Superconducting planes, however, are unstable against the formation of a system of Abrikosov vortices between them. The distance  $L$  between the vortices may be found from the condition  $L\nabla\phi = \pi$ . In the case of superconducting filaments, such boundary vortices cannot be formed because the phase difference  $\Delta\phi$  along the filament–filament boundary is much less than  $\pi$ .



**Figure 3.** Profiles of the order parameter for different values of  $\psi(0)$  and  $H_0/H_{c2} = 0.9$ . In this case, as well as for a single filament, these profiles are independent of  $\kappa$  for  $10 < \kappa < 100$ . The inset shows the total free energy of the system of filaments  $F = n\tilde{F}_{fil}$  as a function of their density. The solid curve is a guide to the eye.

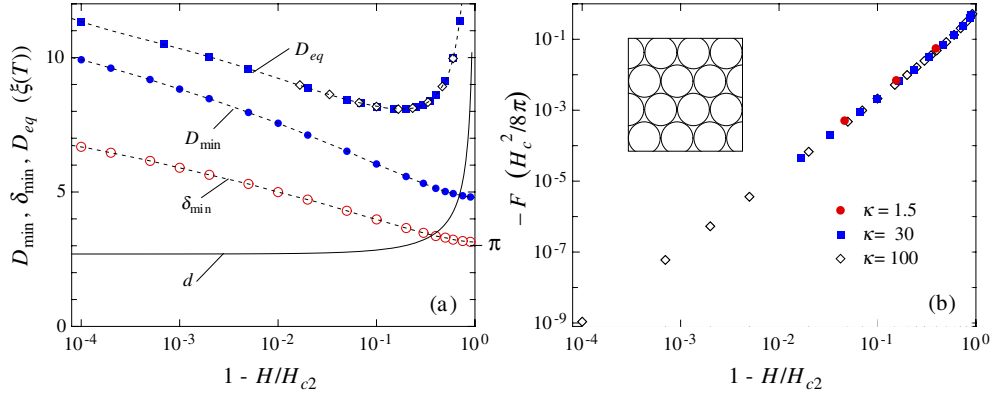
Because the number of filaments in the sample is only limited by a short-range repulsion, they are expected to form a rather dense triangular configuration as is illustrated in the inset to figure 4(b). In order to analyse the resulting profile of an individual filament, we again use equations (1) and (2) but, in order to satisfy the condition  $\psi = 0$  between the filaments, the boundary condition  $d\psi/dr|_{r \rightarrow \infty} = 0$  is abandoned. Several solutions of the GL equations are shown in figure 3. In this case, the order parameter vanishes with a non-zero derivative and we may introduce the radius of the filament as the value of  $r = D/2$  at the point where  $\psi = 0$ .

As may be seen in figure 3, the diameter  $D$  of the filament decreases with decreasing value of  $\psi(0)$ . The important consequence of the GL equations is that the diameter  $D$  of the filament cannot be smaller than a certain minimal value  $D_{min}$ . The only solution of the GL equations for  $D \leq D_{min}$  is  $\psi(r) \equiv 0$ . The dependence of  $D_{min}$  on  $(1 - H_0/H_{c2})$  is shown in figure 4(a). We note that also for superconducting lamellae a minimal thickness  $\delta_{min}$  exists. The dependence of  $\delta_{min}$  on the applied magnetic field is shown in figure 4(a), as well. Both  $D_{min}$  and  $\delta_{min}$  decrease with decreasing magnetic field. It has been shown that  $\delta_{min}|_{H_0=0} = \pi\xi(T)$  [3]. The corresponding value of  $D_{min}|_{H_0=0} \approx 4.81\xi(T)$ .

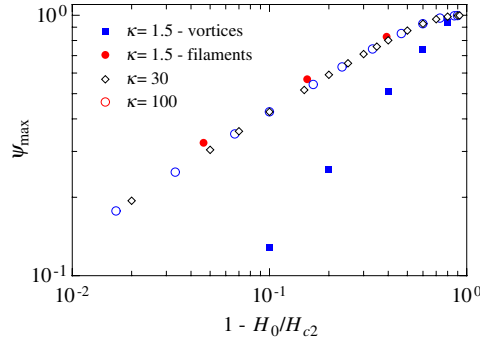
In the following we assume that the distance between the filaments is equal to their diameter  $D$ , as is shown in the inset to figure 4(b). For a triangular lattice, the density  $n$  of filaments is thus  $n = 2/(\sqrt{3}D^2)$ . In order to evaluate the equilibrium density of filaments, we plot the total free energy of the system of filaments  $F = n\tilde{F}_{fil}$ , where  $\tilde{F}$  is the Gibbs free energy calculated for solutions of the type shown in figure 3, versus  $n$  as is illustrated in the inset to figure 3. The position of the minimum on this curve corresponds to the equilibrium value of  $n$ . The magnetic field dependence of the equilibrium diameter  $D_{eq}$  of the filaments is presented in figure 4(a). As may be seen,  $D_{eq}$ , which is the period of the triangular lattice of the filaments, is considerably larger than the equivalent quantity  $d$  for the vortex structure.

The dependencies of  $\psi(0)$  and  $(1 - H_0/H_{c2})$  on the magnetic field for an equilibrium filament configuration practically coincide with those calculated in the single-filament approximation for  $H_0/H_{c2} \geq 0.5$  (see figures 2(a) and (b)).

Figure 4(b) shows the free energy of the system of filaments as a function of  $(1 - H_0/H_{c2})$ . The energy was calculated assuming that the filaments are cylindrical. We have to adopt



**Figure 4.** (a) The minimal thickness of superconducting lamellae,  $\delta_{\min}$ , the minimal diameter of the superconducting filament,  $D_{\min}$ , and the equilibrium value  $D_{eq}$  versus  $(1 - H_0/H_{c2})$ . The period  $d$  of the vortex structure is shown as the solid curve. The dashed curves are guides to the eye. (b) The Gibbs free energy  $F$  of the system of filaments (per unit of volume) with respect to that of the normal metal in a magnetic field equal to  $H_0$ . The inset represents the triangular lattice of superconducting filaments.



**Figure 5.** The maximum amplitude of the order parameter for the equilibrium configuration of filaments and for the vortex lattice. The data for the vortices is taken from [4].

this approach, in order to use one-dimensional GL equations which are the basis of our consideration. It is obvious, however, that, because of their mutual interaction, the filaments should adopt a hexagonal rather than a circular cross-section. We note, however, that this simplification may only result in free-energy values that are slightly higher than the actual ones.

As we have argued, the system of quantized vortices is not the only way to realize the mixed state of type-II superconductors, which may also be established by an ensemble of superconducting filaments. At low magnetic fields, where the equilibrium diameter of filaments  $D_{eq} \geq \lambda(T)$ , the filaments are unstable against the formation of a vortex line in the centre of the filament. In order to identify the stable arrangement in higher fields, the free energies for these two configurations should be compared. To our knowledge the only study in which the two-dimensional GL equations have been solved for the vortex lattice is presented in [4]. Figure 5 shows the results of our calculations of the order parameter amplitude  $\psi_{\max}$  versus  $(1 - H_0/H_{c2})$  for filaments, together with the results of [4]. In the high-magnetic-field limit, the term proportional to the magnetic moment in the Gibbs free energy may be neglected and the

density of the free energy reduces to  $F \approx -\psi^4(r)H_c^2/8\pi$ . As may clearly be seen in figure 5, the amplitude of  $\psi$  for the vortex lattice is considerably smaller than that for the filaments and the ratio  $\psi_{\max}(\text{filaments})/\psi_{\max}(\text{vortices})$  increases with increasing magnetic field. Thus, in sufficiently high magnetic fields, the free energy for the configuration of superconducting filaments is expected to be lower than that for the vortex lattice.

The properties of a mixed state consisting of superconducting filaments are quite different from those of the Abrikosov vortices. First, because the filaments are always separated by normal-conducting regions, the sample resistance for currents perpendicular to the direction of the magnetic field never vanishes and the true zero-resistance superconducting state may be achieved only after the transition to the vortex structure. Second, in the case of filaments, the magnetic flux faces no barriers to moving in or out of the sample and the magnetization of the sample must be reversible, independent of whether the filaments are pinned or not.

The analysis presented in this letter shows that, in magnetic fields close to  $H_{c2}$ , the mixed state may well consist of a triangular lattice of superconducting filaments separated by regions where the superconducting order parameter  $\psi = 0$ . With decreasing external magnetic field, the configuration of superconducting filaments necessarily has to undergo a transition to the conventional mixed state, involving Abrikosov vortices. The value of the transition field is determined by the free-energy balance between these two configurations which cannot be determined without more precise calculations of the free energy for both cases. The transition from one type of mixed state to the other involves a complete change of topology and must be accompanied by discontinuities in both the resistivity and the magnetic moment of the sample. We also expect some hysteresis, as well as a latent heat, dictated by the discontinuity of the magnetization. In other words, this transition is expected to exhibit all the features of a first-order phase transition. Such transitions are observed in high-temperature superconductors at  $H < H_{c2}$  and are usually attributed to the melting of the vortex lattice [5–8].

We wish to thank R Monnier for numerous stimulating discussions.

## References

- [1] Abrikosov A A 1988 *Fundamentals of the Theory of Metals* (Amsterdam: North-Holland)
- [2] Cossy-Favre A, Dutoit B, Holguin E, Landau I and Rinderer L 1994 *J. Low Temp. Phys.* **94** 619
- [3] Andreev A F and Tekel P 1972 *Zh. Eksp. Teor. Fiz.* **76** 1740 (Engl. transl. 1972 *Sov. Phys.-JETP* **35** 807)
- [4] Brandt E H 1997 *Phys. Rev. Lett.* **78** 2208
- [5] Pastoriza H, Goffman M F, Arribère A and de la Cruz F 1994 *Phys. Rev. Lett.* **72** 2951
- [6] Liang R, Bonn D A and Hardy W N 1996 *Phys. Rev. Lett.* **76** 835
- [7] Welp U, Fendrich J A, Kwok W K, Crabtree G W and Veal B W 1996 *Phys. Rev. Lett.* **76** 4809
- [8] Schilling A, Fisher R A, Phillips N E, Welp U, Dasgupta D, Kwok W K and Crabtree G W 1996 *Nature* **382** 701
- [9] Roulin R, Junod A and Walker E 1996 *Science* **273** 1210